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SOLUTION BY THE PROPOSER.

Let a = number of letters and spaces in heading.

b = number of spaces on 80 scale of right stop.

Then $\frac{80 - a}{2}$ = number on 80 scale to begin heading. But by the method

used this number is found as $\left\{ \left(40 - \frac{b}{2} \right) - \frac{a}{2} \right\} + \frac{b}{2}$.

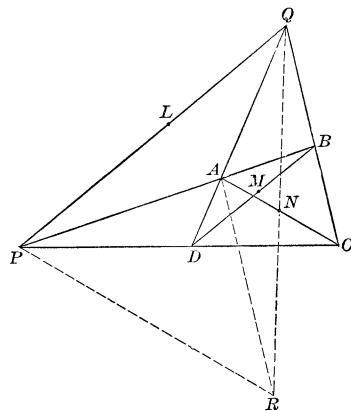
GEOMETRY.

418. Proposed by MRS. H. E. TREFETHEN, Waterville, Maine.

The difference of the squares of the two interior diagonals of a cyclic quadrilateral is to twice their rectangle as the distance between their mid-points is to the third diagonal.

SOLUTION BY C. N. SCHMALL, New York, N. Y.

Let $ABCD$ be a cyclic quadrilateral. Let the sides BA, CD meet in P , and the sides DA, CB in Q . Draw PQ . The resulting figure is a *complete quadrilateral*.



lateral and PQ is the *third diagonal*. Let L, M, N , be the mid-points of the three diagonals. Then we have to prove that

$$\frac{\overline{AC}^2 - \overline{BD}^2}{2\overline{AC} \cdot \overline{BD}} = \frac{MN}{PQ}.$$

It is well known that the points, L, M, N , are collinear. (See RUSSELL'S *Sequel to Elem. Geom.*, p. 154, ex. 5; or LACHLAN'S *Pure Geom.*, p. 93, § 151; or MILNE'S *Cross-Ratio Geom.*, p. 113, ex. 3.) Hence $MN = LN - LM$, or

$$\frac{MN}{PQ} = \frac{LN}{PQ} - \frac{LM}{PQ}. \quad (1)$$

Draw QN and prolong it to R , making $NR = QN$. Draw AR . Then $QCRA$

is a parallelogram. Hence QC is parallel to AR and

$$QC = AR. \quad (2)$$

Now, the triangles PAC, PDB , are similar; for $\angle PCA = \angle PBD$, both standing on the arc AD in a cyclic quadrilateral. Hence

$$AC : DB :: PA : PD. \quad (3)$$

Likewise, the triangles QCA, QDB are similar; and

$$AC : DB :: QC : QD. \quad (4)$$

From (3) and (4),

$$PA : PD :: QC : QD;$$

or

$$PD : QD :: PA : QC;$$

or, by (2),

$$PD : QD :: PA : AR. \quad (5)$$

Again,

$$\angle PDQ = \angle PAR, \quad (6)$$

for

$$\angle PDQ = 180^\circ - \angle ADC = \angle ABC$$

and

$$\angle PAR = \angle ABC.$$

Hence, by (5) and (6) the triangles PDQ and PAR are similar; since they have an angle in each equal and the including sides proportional. Therefore,

$$PA : PD :: PR : PQ;$$

or

$$PA : PD :: 2LN : PQ. \quad (7)$$

By (3) and (7) we have

$$AC : BD :: 2LN : PQ. \quad (8)$$

In a similar way, by drawing QM and prolonging it to S (say), making $MS = QM$, and then drawing PS , we can show that

$$AC : BD :: PQ : 2LM. \quad (9)$$

From (8) and (9),

$$\frac{LN}{PQ} = \frac{1}{2} \frac{AC}{BD}, \quad \frac{LM}{PQ} = \frac{1}{2} \frac{BD}{AC}.$$

Substituting these values in (1), we have

$$\frac{MN}{PQ} = \frac{1}{2} \left(\frac{AC}{BD} - \frac{BD}{AC} \right); \quad \text{or} \quad \frac{MN}{PQ} = \frac{\overline{AC}^2 - \overline{BD}^2}{2AC \cdot BD}.$$

CALCULUS.

333. Proposed by C. N. SCHMALL, New York City.

Evaluate $\int \int \int \sqrt{\frac{1 - (x^2 + y^2 + z^2)}{1 + x^2 + y^2 + z^2}} dx dy dz$, where $x^2 + y^2 + z^2 < 1$.